A high-resolution coupled permafrost model

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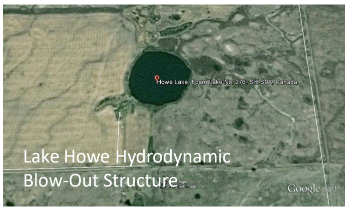
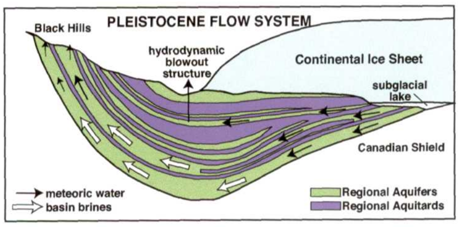
Motivation

- Permafrost in advance to glaciation can
  - Influence the dynamics of glaciation, mainly by delaying warm-bed conditions
  - Hence also timings
  - Re-define basal and bedrock hydrology
- Currently main application:
  - Nuclear waste repository safety assessment (POSIVA, NAGRA)



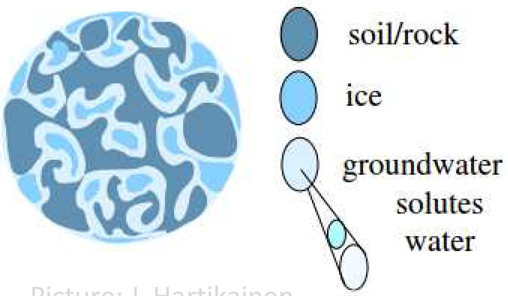


Grasby et al. (2000)

Permafrost model

- **Saturated porous medium that consists of skeleton of rock or soil, ice and groundwater of water and dissolved salts** :

1. Heat transfer
2. Groundwater flow of saturated aquifer (Darcy)
3. Solute transport within groundwater
4. Deformation of bedrock (porosity)

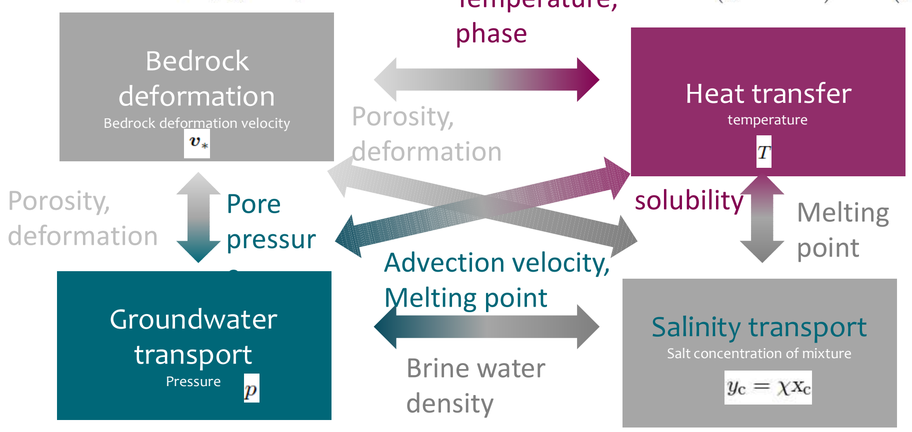


Picture: J. Hartikainen

Permafrost model

$$-\mathrm{div}\left(\boldsymbol{\sigma}^{\mathrm{ef}} - p\mathbf{I}\right) = \rho_{\mathrm{G}}\boldsymbol{g},$$
$$\boldsymbol{\sigma}^{\mathrm{ef}} = \mathbb{C}_{\mathrm{G}} : \boldsymbol{\epsilon} - \boldsymbol{\beta}_{\mathrm{G}}(T - T_0)$$

$$C_{\mathrm{G}}^{TT}\left(\frac{\partial T}{\partial t} + \boldsymbol{v}_* \cdot \mathrm{grad}\, T\right) + C_{\mathrm{gw}}^{TT}\,\mathrm{grad}\, T \cdot \boldsymbol{J}_{\mathrm{gw}}^{\mathrm{D}} + \mathrm{div}\, \boldsymbol{J}_{\mathrm{G}}^{\mathrm{H}} + $$
$$+ C_{\mathrm{G}}^{Tp}\left(\frac{\partial p}{\partial t} + \boldsymbol{v}_* \cdot \mathrm{grad}\, p\right) + C_{\mathrm{G}}^{Ty_{\mathrm{c}}}\left(\frac{\partial T}{\partial t} + \boldsymbol{v}_* \cdot \mathrm{grad}\, y_{\mathrm{c}}\right) = S_{\mathrm{G}}$$



$$C_{\mathrm{gw}}^{pp}\left(\frac{\partial p}{\partial t} + \boldsymbol{v}_* \cdot \mathrm{grad}\, p\right) + \mathrm{div}\left(\varrho_{\mathrm{gw}}\boldsymbol{J}_{\mathrm{gw}}^{\mathrm{D}}\right) + C_{\mathrm{gw}}^{pT}\left(\frac{\partial T}{\partial t} + \boldsymbol{v}_* \cdot \mathrm{grad}\, T\right) + $$
$$+ C_{\mathrm{gw}}^{py_{\mathrm{c}}}\left(\frac{\partial y_{\mathrm{c}}}{\partial t} + \boldsymbol{v}_* \cdot \mathrm{grad}\, y_{\mathrm{c}}\right) + \mathrm{div}\left[\eta\left(\varrho_{\mathrm{c}} - \varrho_{\mathrm{w}}\right)\boldsymbol{J}_{\mathrm{c}}^{\mathrm{F}}\right] - C_{\mathrm{gw}}^{pI_1}\left(\frac{\partial I_1}{\partial t} + \boldsymbol{v}_* \cdot \mathrm{grad}\, I_1\right) = S_{\mathrm{gw}}.$$

$$C_{\mathrm{c}}^{y_{\mathrm{c}}y_{\mathrm{c}}}\left(\frac{\partial y_{\mathrm{c}}}{\partial t} + \boldsymbol{v}_* \cdot \mathrm{grad}\, y_{\mathrm{c}}\right) + \mathrm{div}\left(\frac{y_{\mathrm{c}}}{\chi}\varrho_{\mathrm{c}}\boldsymbol{J}_{\mathrm{gw}}^{\mathrm{D}}\right) + \mathrm{div}\left(\eta\varrho_{\mathrm{c}}\boldsymbol{J}_{\mathrm{c}}^{\mathrm{F}}\right) + $$
$$+ C_{\mathrm{c}}^{y_{\mathrm{c}}T}\left(\frac{\partial T}{\partial t} + \boldsymbol{v}_* \cdot \mathrm{grad}\, T\right) + C_{\mathrm{c}}^{y_{\mathrm{c}}p}\left(\frac{\partial p}{\partial t} + \boldsymbol{v}_* \cdot \mathrm{grad}\, p\right) = S_{\mathrm{c}}$$

Permafrost model

- Implemented in the open-source software **Elmer**
  - Finite Element Method (standard Galerkin)
  - Easily coupled with **Elmer/Ice** ice-sheet simulation
  - Extra module with about 7000 lines of code
  - Main motivation: MPI/OpenMP parallelism (up to 1000's of cores)
- Each physical problem implemented in separate module
  - Fixed-point iterations on all other variables
- Residual free bubbles for stabilization in case of advection
  - Potential to vectorise algorithm
- Dependent material parameters evaluated at IP's
  - Needed for accuracy
- p-elements of different orders

Heat transfer

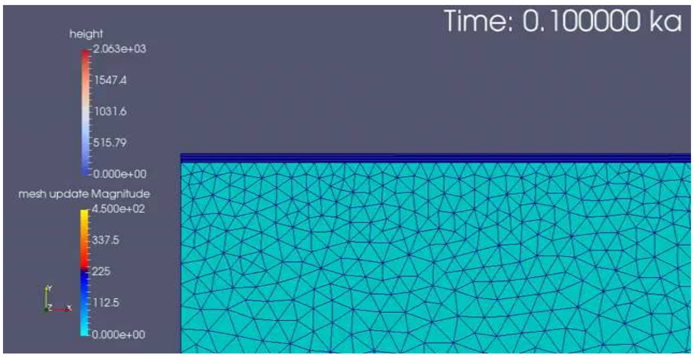
Groundwater transport

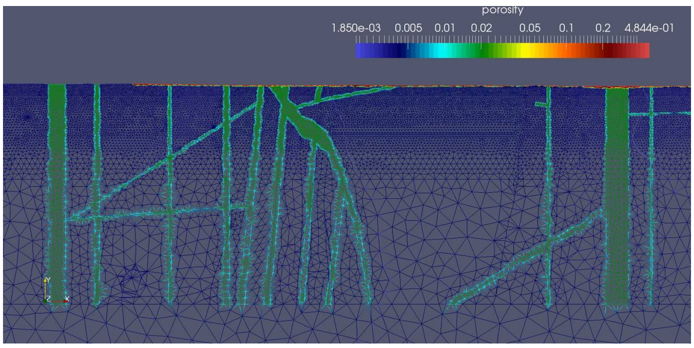
Salinity transport

Bedrock deformation

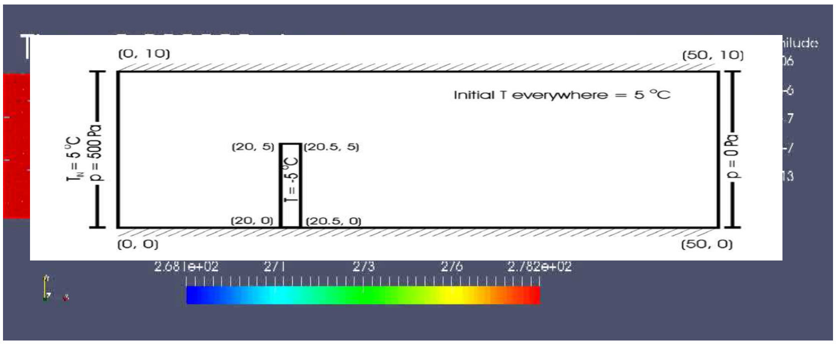
Permafrost Model

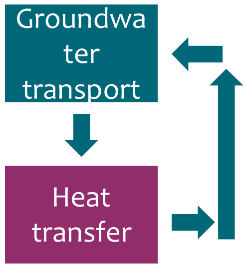
- Multiple bodies
- Different mesh-concepts:
  - **Ice-sheet:** structured, layered mesh
  - **Bedrock:** unstructured, in places high-resolution mesh
  - Offset for displacement: Model for glacial isostatic adjustment (LLRA)





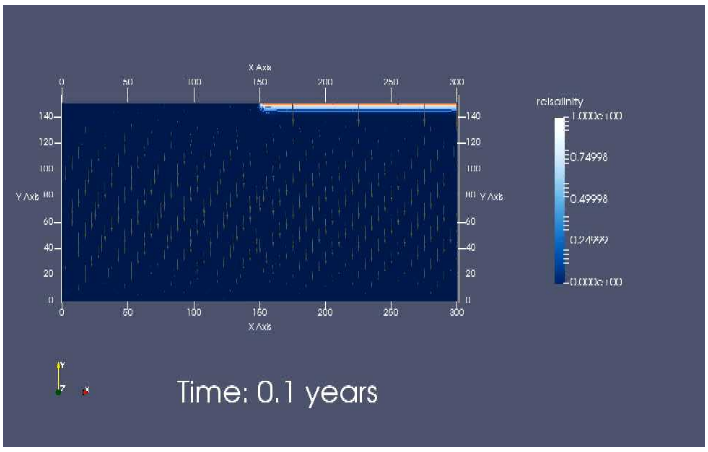
Validation of single components

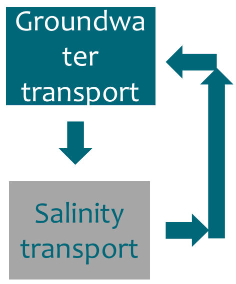




Coupled groundwater flow (after McKenzie at al., 2007)

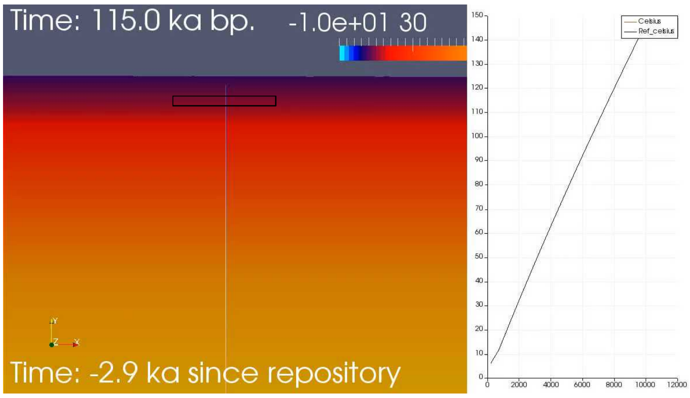
Validation of single components





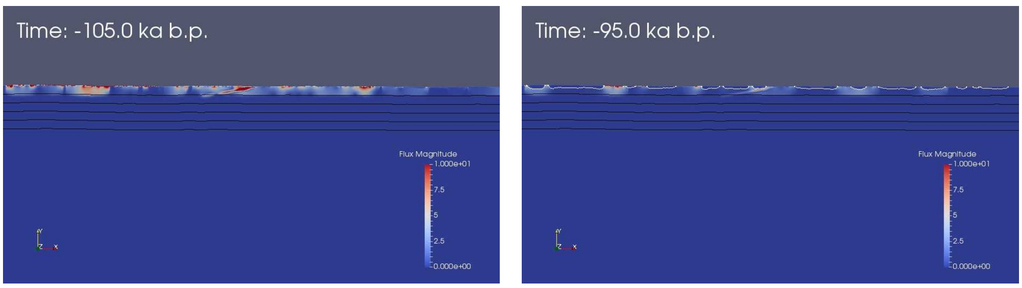
Elder Problem (Voss and Souza, 1987): salinity transport in porous medium

Real world example: Forsmark



- Test case for nuclear waste repository at ~450 m depth
  - High-res cross-section mesh
- Includes full model:
  - Heat transfer + phase change
  - Darcy flow
  - Solute transport
- Forcing with climate data 115ka – 70 ka bp
  - White line shows permafrost boundary
  - Nuclear repository at 3ka

Real world example: Forsmark



Change of near-surface groundwater flux with permafrost

Outlook

- Implementation in 3D problems (large parallel) – challenge in pre-processing
- Process studies at glacier bedrock (sliding laws)
- Finalizing bedrock deformation model (based on existing linear elasticity solver)
- GIA model (global translational velocity)
- Adding physical surface model (BC's for pressure, temperature and Salinity)
- Improving solver performance using OpenMP (multi-threading, SIMD)

End of talk – thank you!

http://www.csc.fi/elmer

Additional Material

Permafrost model

(T, p, y_c, η)

- Heat Transfer:

$$C_G^{TT}\left(\frac{\partial T}{\partial t} + \boldsymbol{v}_* \cdot \operatorname{grad} T\right) + C_{gw}^{TT} \operatorname{grad} T \cdot \boldsymbol{J}_{gw}^{D} + \operatorname{div} \boldsymbol{J}_G^{H} + \\ + C_G^{Tp}\left(\frac{\partial p}{\partial t} + \boldsymbol{v}_* \cdot \operatorname{grad} p\right) + C_G^{Ty_c}\left(\frac{\partial T}{\partial t} + \boldsymbol{v}_* \cdot \operatorname{grad} y_c\right) = S_G$$

- Groundwater flow:

$$C_{gw}^{pp}\left(\frac{\partial p}{\partial t} + \boldsymbol{v}_* \cdot \operatorname{grad} p\right) + \operatorname{div}\left(\varrho_{gw} \boldsymbol{J}_{gw}^{D}\right) + C_{gw}^{pT}\left(\frac{\partial T}{\partial t} + \boldsymbol{v}_* \cdot \operatorname{grad} T\right) + \\ + C_{gw}^{py_c}\left(\frac{\partial y_c}{\partial t} + \boldsymbol{v}_* \cdot \operatorname{grad} y_c\right) + \operatorname{div}\left[\eta\left(\varrho_c - \varrho_w\right) \boldsymbol{J}_c^{F}\right] - C_{gw}^{pI_1}\left(\frac{\partial I_1}{\partial t} + \boldsymbol{v}_* \cdot \operatorname{grad} I_1\right) = S_{gw}.$$

- Solute transport:

$$C_c^{y_c y_c}\left(\frac{\partial y_c}{\partial t} + \boldsymbol{v}_* \cdot \operatorname{grad} y_c\right) + \operatorname{div}\left(\frac{y_c}{\chi} \varrho_c \boldsymbol{J}_{gw}^{D}\right) + \operatorname{div}\left(\eta \varrho_c \boldsymbol{J}_c^{F}\right) + \\ + C_c^{y_c T}\left(\frac{\partial T}{\partial t} + \boldsymbol{v}_* \cdot \operatorname{grad} T\right) + C_c^{y_c p}\left(\frac{\partial p}{\partial t} + \boldsymbol{v}_* \cdot \operatorname{grad} p\right) = S_c$$

- Bedrock deformation:

$$-\operatorname{div}\left(\boldsymbol{\sigma}^{\text{ef}} - p\mathbf{I}\right) = \rho_G \boldsymbol{g},$$

$$\boldsymbol{\sigma}^{\text{ef}} = \mathbb{C}_G : \boldsymbol{\epsilon} - \boldsymbol{\beta}_G (T - T_0)$$

Permafrost model

- Heat flux:

$$\boldsymbol{J}_{\mathrm{G}}^{\mathrm{H}} = -\mathbf{K}_{\mathrm{G}}^{TT} \cdot \operatorname{grad} T$$

- Groundwater flux:

$$\boldsymbol{J}_{\mathrm{gw}}^{\mathrm{D}} = -\left(\mathbf{K}_{\mathrm{gw}}^{pp} \cdot \operatorname{grad} p - \mathbf{K}_{\mathrm{gw}} \cdot \varrho_{\mathrm{gw}} \boldsymbol{g} + \mathbf{K}_{\mathrm{gw}}^{pT} \cdot \operatorname{grad} T\right).$$

- Solute flux:

$$\boldsymbol{J}_{\mathrm{c}}^{\mathrm{F}} = -\left(\mathbf{K}_{\mathrm{c}}^{y_{\mathrm{c}} y_{\mathrm{c}}} \cdot \operatorname{grad} y_{\mathrm{c}} - \mathbf{K}_{\mathrm{c}} \cdot \boldsymbol{f}_{\mathrm{c}}\, y_{\mathrm{c}}\right)$$

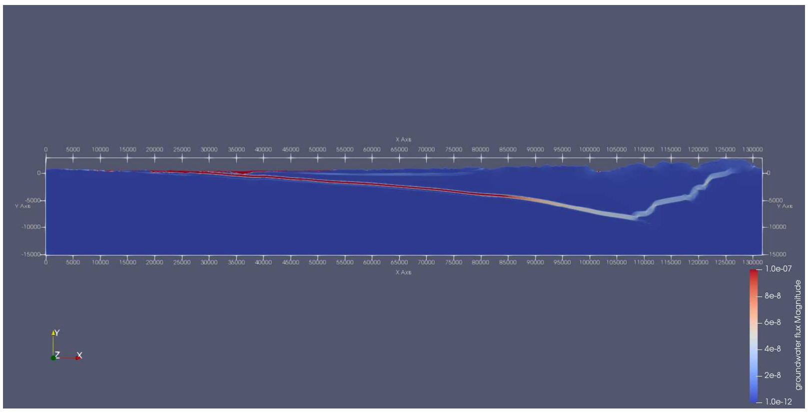
- Elasticity tensor:

$$\mathbf{K}_{\mathrm{G}}^{\boldsymbol{uu}} = \frac{E_{\mathrm{G}}}{(1+\nu_{\mathrm{G}})(1-2\nu_{\mathrm{G}})} \begin{bmatrix} 1-\nu_{\mathrm{G}} & \nu_{\mathrm{G}} & \nu_{\mathrm{G}} & 0 & 0 & 0 \\ \nu_{\mathrm{G}} & 1-\nu_{\mathrm{G}} & \nu_{\mathrm{G}} & 0 & 0 & 0 \\ \nu_{\mathrm{G}} & \nu_{\mathrm{G}} & 1-\nu_{\mathrm{G}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu_{\mathrm{G}}}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu_{\mathrm{G}}}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu_{\mathrm{G}}}{2} \end{bmatrix},$$

$$E_{\mathrm{G}} = (1-\eta)\frac{E_{\mathrm{s},0}}{1-\eta_0} + \eta(1-\chi)E_{\mathrm{i}},$$

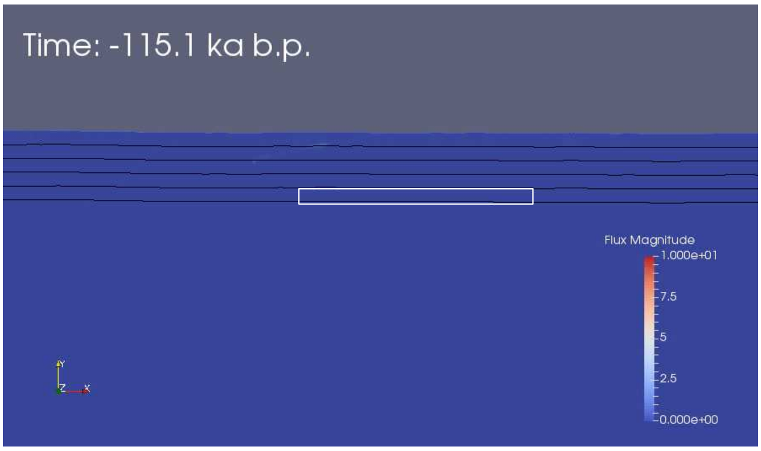
$$\nu_{\mathrm{G}} = (1-\eta)\nu_{\mathrm{s},0} + \eta(1-\chi)\nu_{\mathrm{i}}.$$

Real world example: Rhine Glacier



Groundwater flux at Rhine-glacier

Real world example: Forsmark



Change of near-surface groundwater flux with permafrost