

Gas Tungsten Arc Welding heat transfer simulation

1. The GTAW Heat transfer problem:

The GTA Welding or TIG (Tungsten Inert Gas) process is studied in a simple case involving only the Heat transfer physics. This document describes the coefficients used in the state equation and its boundary conditions in order to understand the gtaw.sif attached to the post (as well as the mesh files...). This is just a preliminary work. I hope to add the Navier-Stokes, the electromagnetism equations in order to take into account the Lorentz force and the free surface deformation on the weld-pool surface. Figure 1, here below, depicts all the phenomena involved in GTA Welding.

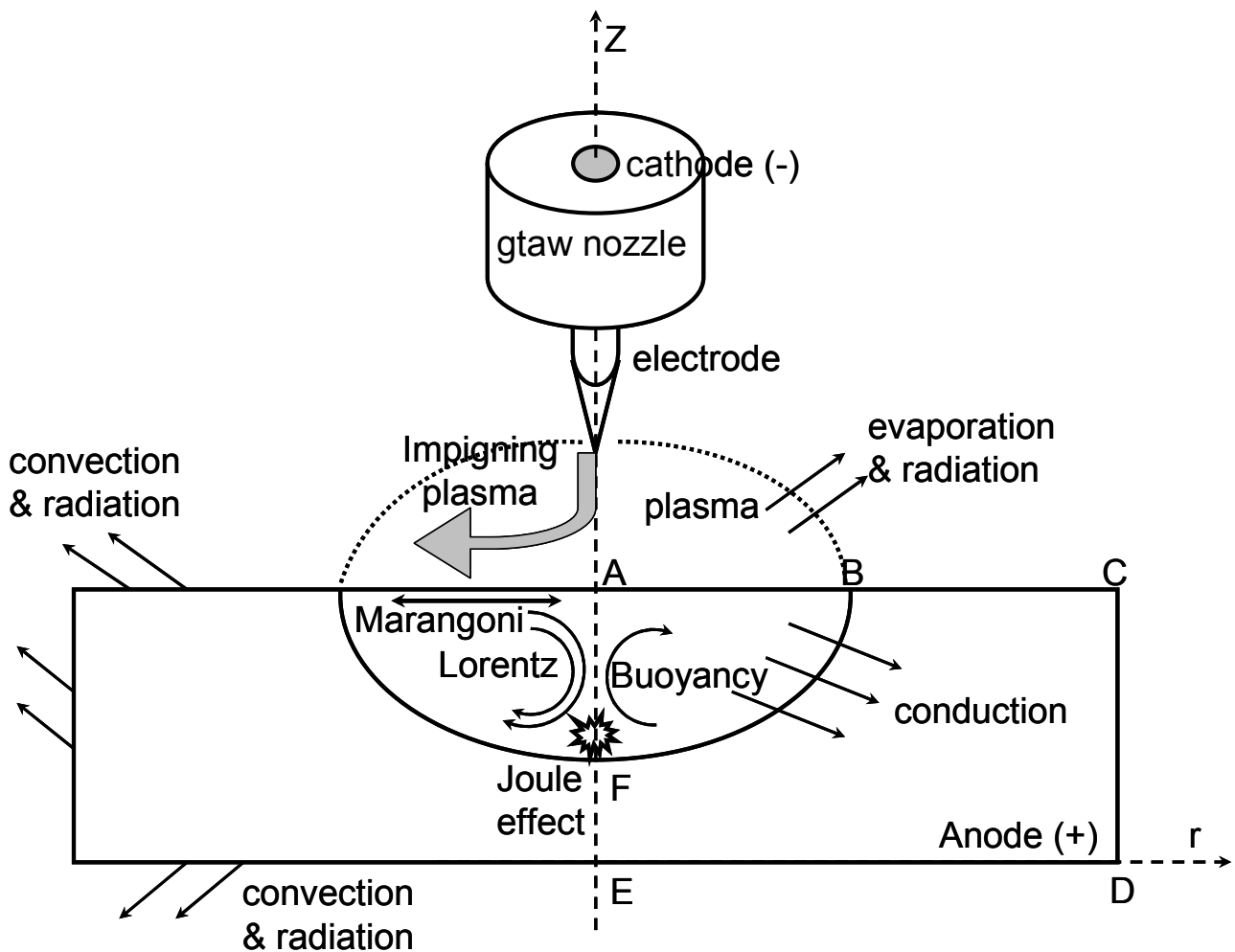


Figure 1: magneto-hydrodynamic problem involved in GTAW.

In order to simplify the problem, the following assumptions are considered:

- The study is restricted to GTA spot welding, => axis ymmetric coordinate system, figure 2.
- No fluid flow;
- No electromagnetism.

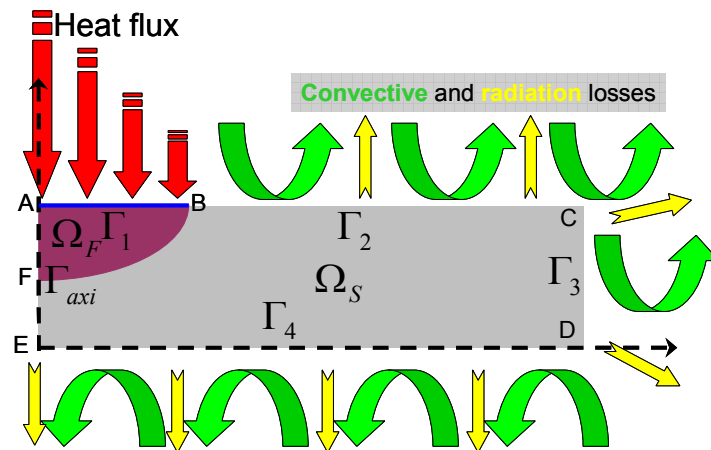


Figure 2 : axisymmetric domain and heat transfer modelling.

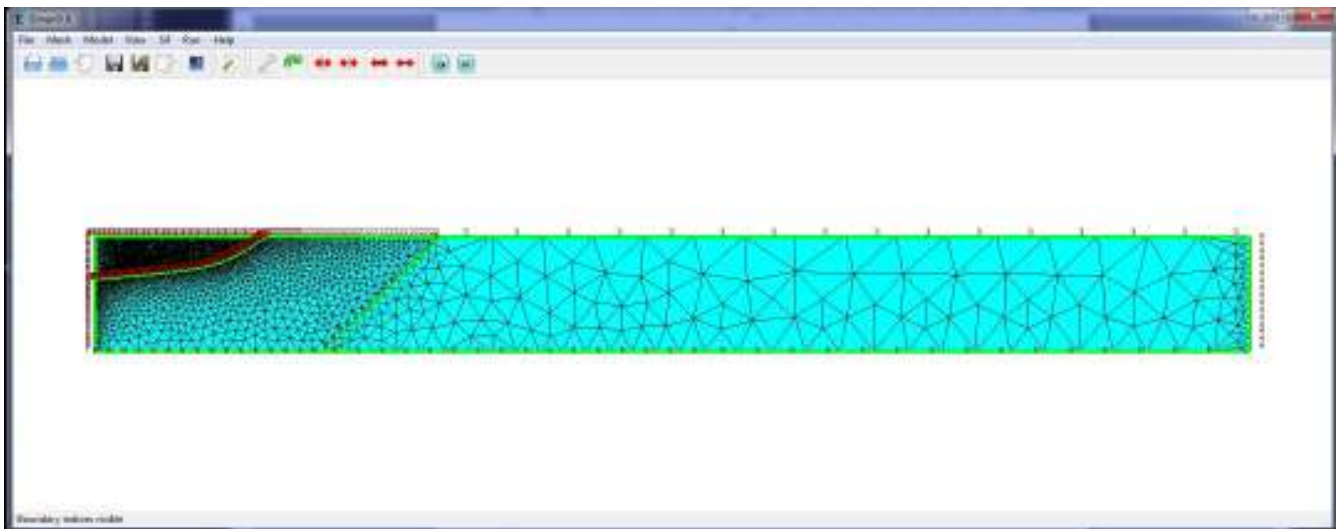


Figure 3: mesh used (radius = 20mm and thickness = 4 mm). On the top, the boundary numbers are 1, 2 and 3; the right vertical is 4, the bottom ones are 5 and 6. The left vertical (symmetry axis) are 7 and 8. the inner ones are (from center to outward) 10 and 9.

THE HEAT TRANSFER:

- **Energy conservation (for the computation of the temperature T):**

$$\rho C_p^{eq} \frac{\partial T}{\partial t} - \frac{1}{r} \frac{\partial}{\partial r} \left(r \lambda \frac{\partial T}{\partial r} \right) - \frac{\partial}{\partial z} \left(\lambda \frac{\partial T}{\partial z} \right) = 0$$

With T is the temperature field. ρ , C_p and λ are respectively the mass density, specific heat and thermal conductivity of the Stainless Steel Metal.

The heat transfer boundary conditions are (it refers to figure 2):

- On the top surface $\Gamma_1 \cup \Gamma_2$: $-\lambda \frac{\partial T}{\partial z} = -\Phi(r, t) + h(T - T_0) + \varepsilon \sigma (T^4 - T_0^4)$
- On the lateral side Γ_3 : $-\lambda \frac{\partial T}{\partial r} = h(T - T_0) + \varepsilon \sigma (T^4 - T_0^4)$
- On the bottom surface Γ_4 : $\lambda \frac{\partial T}{\partial z} = h(T - T_0) + \varepsilon \sigma (T^4 - T_0^4)$
- On the symmetry axis: $-\lambda \frac{\partial T}{\partial r} = 0$
- The initial condition is: $T(r, t = 0) = T_0$

$\Phi(r)$ is a surface heat flux exchanged between the plasma arc and the work-piece. We will assume that this heat flux distribution obey to a Gaussian distribution. So it can be written as follows:

$$\Phi(r, t) = \eta \frac{1}{2} \frac{U_s \cdot I_s}{\pi r_b^2} e^{-\frac{1}{2} \left(\frac{r}{r_b} \right)^2} \quad \text{where } U_s \text{ is the welding tension, } I_s \text{ is the welding intensity, } \eta \text{ is the}$$

GTAW efficiency and r_b is called the Gaussian radius.

The IHTP general formulation is written as:

Find the unknown vector $\bar{p} = \{p_1, \dots, p_M\}$ such that the measured temperature $Y_{mes}(t)$ is equal to the calculated temperature $T_{cal}(x_i, t; \bar{p})$ at each sensors located and at point x_i , $i = 1, \dots, n_s$ and for any time steps.

Generally a second formulation is used: this one introduces a quadratic criterion (also called objective function) [M. N. Necati and H. R. Orlande. Inverse heat transfer, fundamentals and applications. Taylor and Francis, New York (2000)] :

$$S(\bar{p}) = \frac{1}{2} [Y_i - T_i]^T W [Y_i - T_i] \text{ or } S(\bar{p}) = \frac{1}{2} \int_{t_0}^{t_f} \int_{\Omega} \sum_{i=1}^{n_s} (Y_{mes}(t) - T_{cal}(x_i, t; \bar{p}))^2 d\Omega dt$$

where Y_i are the measured temperatures and T_i the calculated temperatures at point x_i , $i = 1, \dots, n_s$. The subscript T denotes the transpose: $[Y_i - T_i]^T = [Y_1 - T_1, \dots, Y_{n_s} - T_{n_s}]$. So, the second formulation of the inverse problem becomes:

Find the vector \bar{p} which minimizes the cost function $S(\bar{p})$.

The Levenberg-Marquardt algorithm is the following (set in python file):

The levenberg-Marquardt method has been widely used in various inverse heat transfer problems. Only some key points of this method are discussed. The Levenberg-Marquardt method associated to an iterative process leads to estimate the unknown vector \bar{p} at iteration $k+1$ as follows:

$$\bar{p}^{k+1} = \bar{p}^k + \left[(J^k)^T W J^k + \lambda^k \Omega^k \right]^{-1} \left[(J^k)^T W (T_i(\bar{p}^k) - Y_i) \right]$$

Where $J(\bar{p})$ is the sensitivity matrix which is defined as the transpose of the term $[\partial T_i(\bar{p}) / \partial p]$. The elements of the sensitivity matrix are called the sensitivity coefficients. The sensitivity coefficient is thus defined as the first derivative of the estimated temperature with respect to the unknown parameters $\bar{p} = \{\eta, R_{B1}, R_{B2}\}$. λ^k is positive scalar which is introduced to alleviate that matrix $J^T J \approx 0$ is ill-conditionned near the initial guess used for the unknown parameters. Ω^k is a diagonal matrix. The term $\lambda^k \Omega^k$ damps oscillations and instabilities due to the ill-conditioned character of the problem. This damping parameter is large at the beginning of the iterative procedure (and the method is like the steepest descent method) then it decreases when the procedure advances to the solution (and the method tends to the Gauss method). W is a diagonal matrix where the diagonal elements are given by the inverse of the standard deviation of the measurement errors.

Suppose that the temperature measurements Y_i are given at each point x_i , $i = 1, \dots, n_s$. We choose an initial set of parameters $\bar{p}^0 = \{\eta, R_{B1}, R_{B2}\}$ and an initial value for the damping parameter $\lambda^0 = 0.001$. The iteration number is initialized ($k = 0$).

- Step 1: Solve the direct problem with the available estimate \bar{p}^k in order to obtain the temperature vector $T_j(\bar{p}^k) = (T_{1,0}, \dots, T_{M,t_f})$.
- Step 2: Compute $S(\bar{p}^k)$.
- Step 3: Compute the sensitivity matrix $J(\bar{p}^k)$ and then set the matrix $\Omega^k = I$.
- Step 4: Calculate the new set of estimate $\bar{p}^{k+1} = \bar{p}^k + \left[(J^k)^T W J^k + \lambda^k \Omega^k \right]^{-1} \left[(J^k)^T W (T_i(\bar{p}^k) - Y_i) \right]$
- Step 5: Solve the direct problem with the new estimate \bar{p}^{k+1} in order to find $T_j(\bar{p}^{k+1})$. Then compute $S(\bar{p}^{k+1})$, as defined in step 2.
- Step 6: if $S(\bar{p}^{k+1}) \geq S(\bar{p}^k)$, replace λ^k by $\lambda^{k+1} = 10 \cdot \lambda^k$ and return to step 4.
- Step 7: if $S(\bar{p}^{k+1}) < S(\bar{p}^k)$, accept the new set of estimate \bar{p}^{k+1} and replace λ^k by $\lambda^{k+1} = \lambda^k / 10$.

Check the stopping criteria. Stop the iterative procedure if it is satisfied; Otherwise, replace k by $k + 1$ and go to step 3.

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S. Rouquette, J. Guo, P. Le Masson. Estimation of the parameters of a Gaussian heat source by the Levenberg-Marquardt method: application to the electronic beam welding. *International Journal of Thermal Science* 46 (2007) 128-138.