Jintegral Solver Written by : Peter Raback,2012

Introduction:

The concept of J-integral has found a significant application in fracture mechanics and its calculation is performed along a contour of length (L), surrounding a crack tip. In Elmer, the J-integral is computed at the boundaries surrounding the crack tip for 2D geometry and the subroutine for this computation is executed as an additional postprocessing tool for the base linear elasticity solver.

Theoretical Formulation:

In fracture mechanics, the J-integral can be defined as a 2-D line integral along a counterclockwise contour, L, surrounding the crack tip. Mathematically, the J-integral is expressed as:

$$J = \int_L \left(W n_x - T_i \frac{\partial u_i}{\partial x} \right) dL \qquad (1)$$

where, strain energy density (W) and components(T_x and T_y) of traction vector (**T**) are expressed as:

$$W = \frac{1}{2}\left(\sigma_{xx}\epsilon_{xx} + \sigma_{yy}\epsilon_{yy} + 2\sigma_{xy}\epsilon_{xy} \right) \qquad (2)$$

$$T_x = \sigma_{xx} n_x + \sigma_{xy} n_y \qquad (3)$$

$$T_y = \sigma_{xy} n_x + \sigma_{yy} n_y \qquad (4)$$

In the above equations, σ_{ij} denotes the stress components, ϵ_{ij} denotes the strain components, n_i denotes the normal vector components and u_i denotes the displacement vector components. For a linear elastic material of elastic modulus E, the relation between J-integral and stress intensity factor for plane stress condition (K_I) can be defined by the following relation:

$$J = \frac{K_I^2}{E}$$

Implementation in Elmer Software:

The Jintegral subroutine needs to be compiled within the test case directory using elmerf90 command and then it can be called as a solver in the solver input file. The command for compilation are:

$elmerf90 -o Jintegral.so Jintegral.f90 (in Linux)

>elmerf90 -o Jintegral.dll Jintegral.f90 (in Windows)

The Jintegral solver is used as an additional solver to the linear elasticity solver (StressSolve). It is used as the solver for postprocessing part i.e. it is called for execution after simulation.

Keywords

Solver solver id
Equation string[Jintegral]
It is the describing name for the solver

Procedure "Jintegral" "Jintegral"
Name of the solver subroutine

Exec Solver [After Simulation]
This means the solver is used during the postprocessing after the linear elasticity solver runs the simulation.

Equation equation id
Plane Stress Logical
If set to *True* , compute the solution according to the plane stress situation, $\sigma_{zz}=0$. Applies only in 2D and this condition is required for computation of J-integral.

Calculate Stress Logical
If set to *True*, the stresses are calculated.

Boundary Condition bc id

Jintegral Logical
If set to *True*, the Jintegral is calculated along the target boundaries.